Prediction of intra- and inter-laminar failure of laminates using non-local damage-enhanced mean-field homogenization

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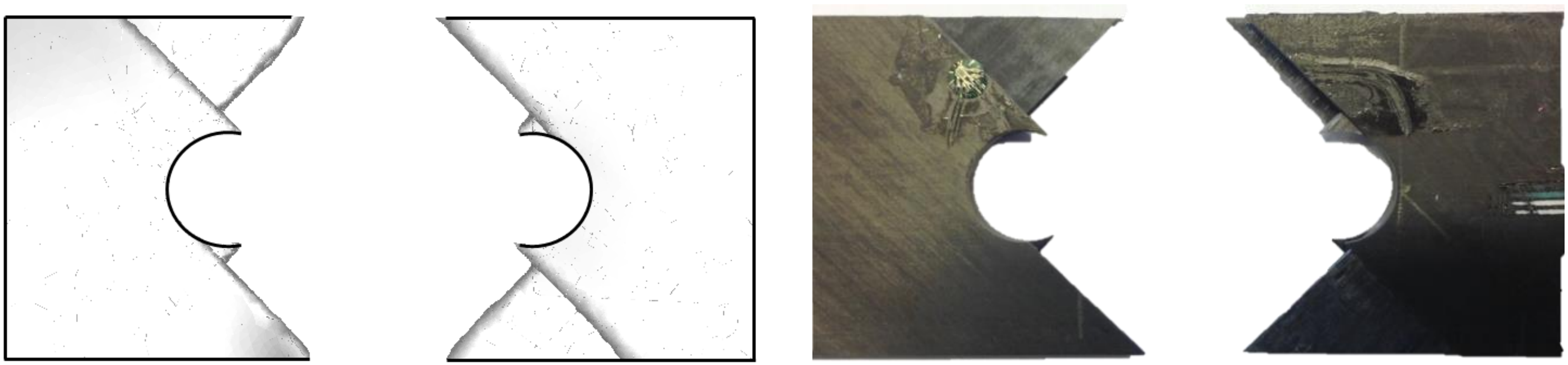

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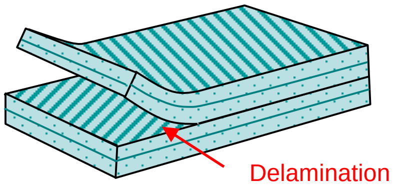
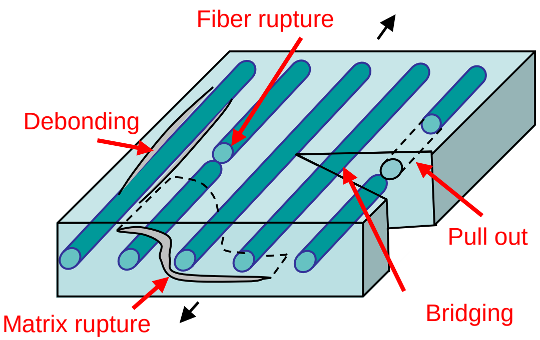
- Multi-scale method for the failure analysis of composite laminates
  - Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization
  - Inter-laminar failure: Hybrid DG/cohesive zone model
  - Experimental validation

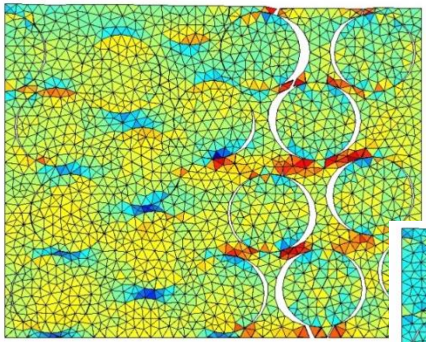
Failure of composite laminates

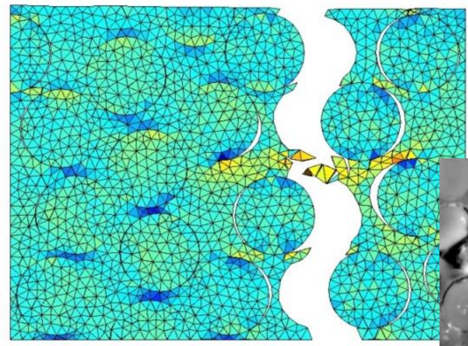
- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation

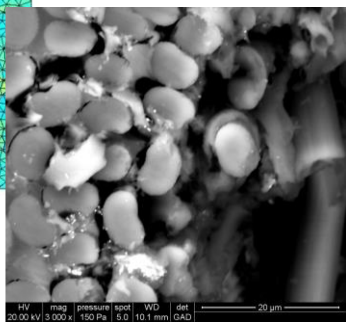




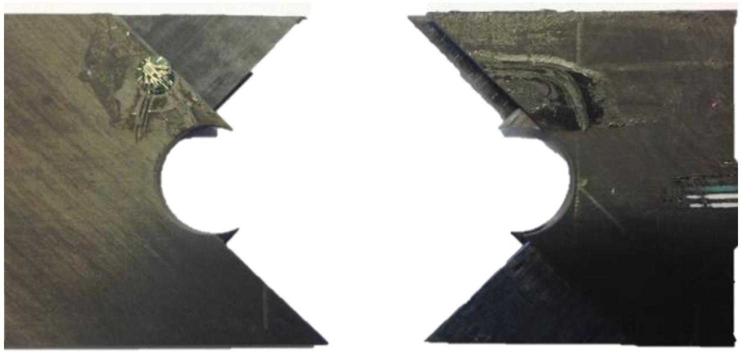
On Micro-scale volume





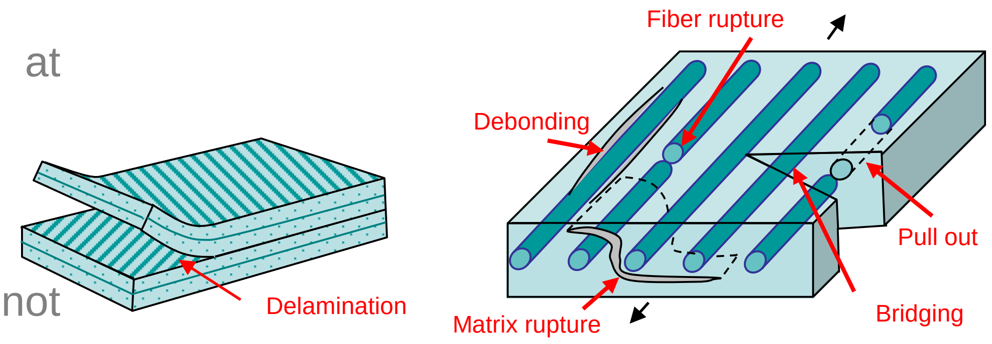


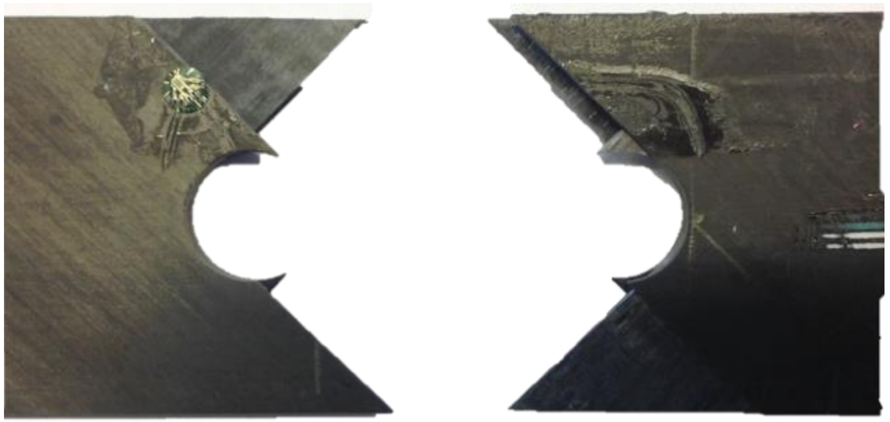
Not possible at structural scale



Failure of composite laminates

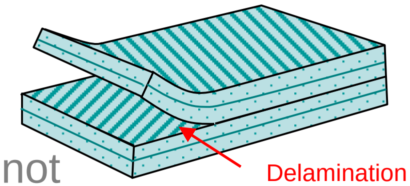
- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation is not possible at structural scale
  - Continuum damage models do not represent accurately the intra-laminar failure
    - Damage propagation direction is not in agreement with experiments

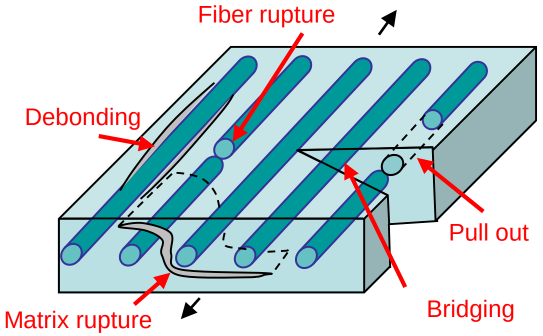


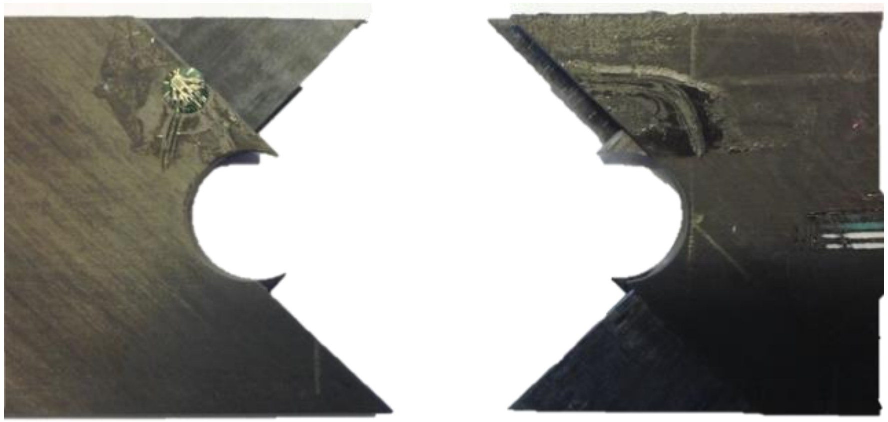


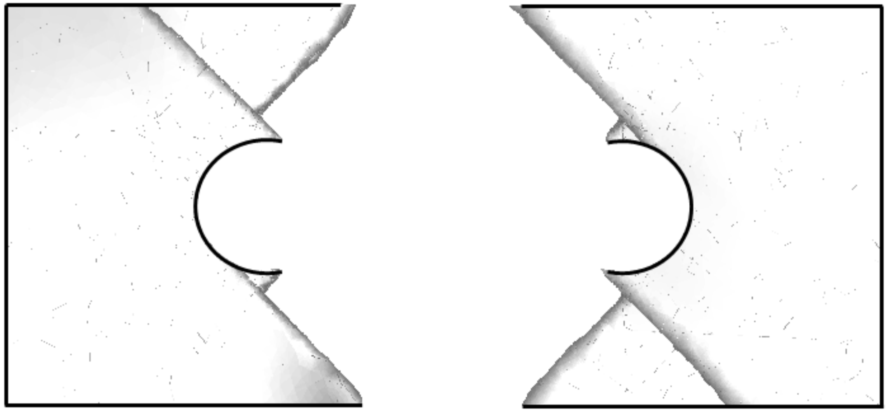
Failure of composite laminates

- Difficulties
  - Different involved mechanisms at different scales
    - Inter-laminar failure
    - Intra-laminar failure
  - Direct finite element simulation is not possible at structural scale
  - Continuum damage models do not represent accurately the intra-laminar failure
    - Damage propagation direction is not in agreement with experiments
- Solution:
  - Embed damage model in a multi-scale formulation
  - For computational efficiency: use of mean-field-homogenization
  - For macro cracks: using hybrid DG/Cohesive zone model



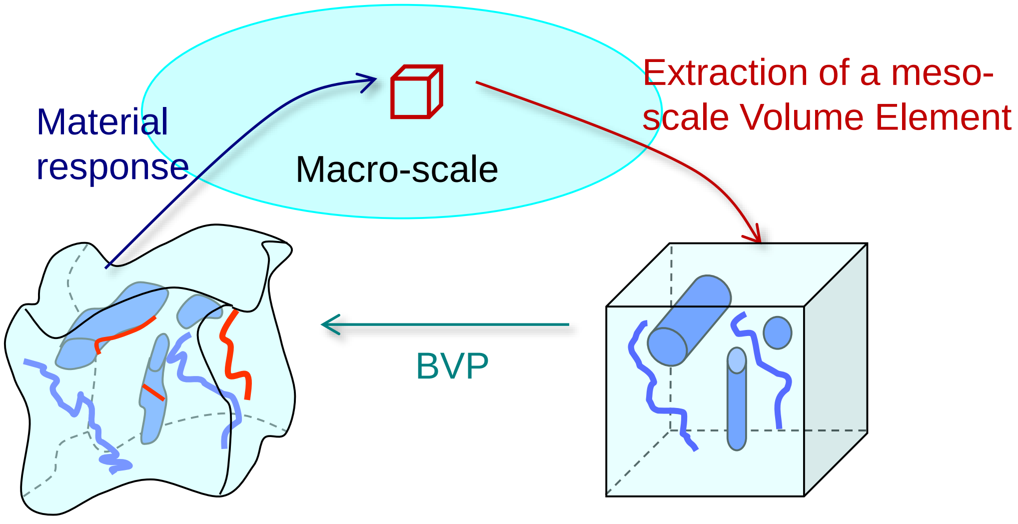






Multi-scale modelling

- Multi-scale modelling
  - One way: homogenization
  - 2 problems are solved (concurrently)
    - The macro-scale problem
    - The meso-scale problem (on a meso-scale Volume Element)



- Length-scales separation

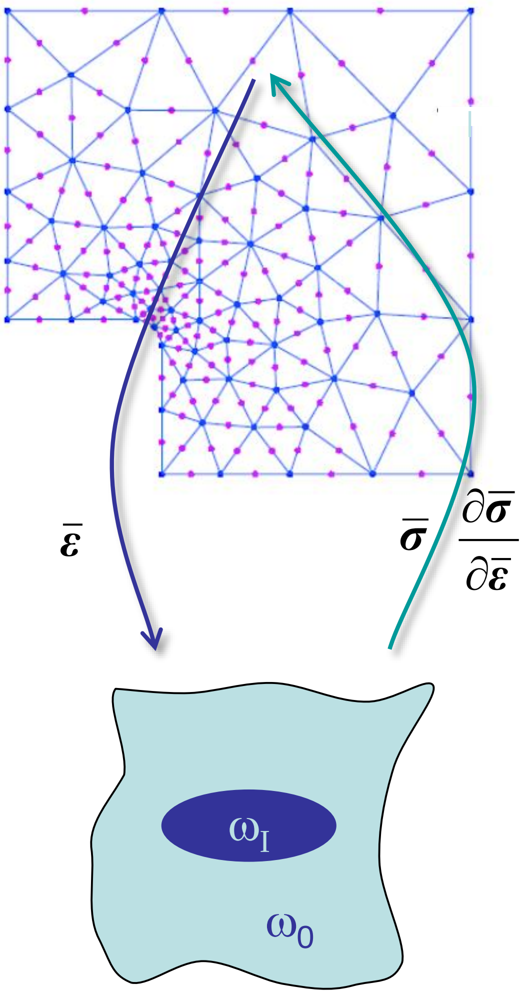
$$L_{macro} >> L_{VE} >> L_{micro}$$

For accuracy: Size of the meso-scale volume element smaller than the characteristic length of the macro-scale loading

To be statistically representative: Size of the meso-scale volume element larger than the characteristic length of the micro-structure

Multi-scale modelling

- Mean-Field-Homogenization
  - Macro-scale
    - FE model
    - At one integration point $\bar{\varepsilon}$ is know, $\bar{\sigma}$ is sought
  - Transition
    - Downscaling: $\bar{\varepsilon}$ is used as input of the MFH model
    - Upscaling: $\bar{\sigma}$ is the output of the MFH model
  - Micro-scale
    - Semi-analytical model
    - Predict composite meso-scale response
    - From components material models



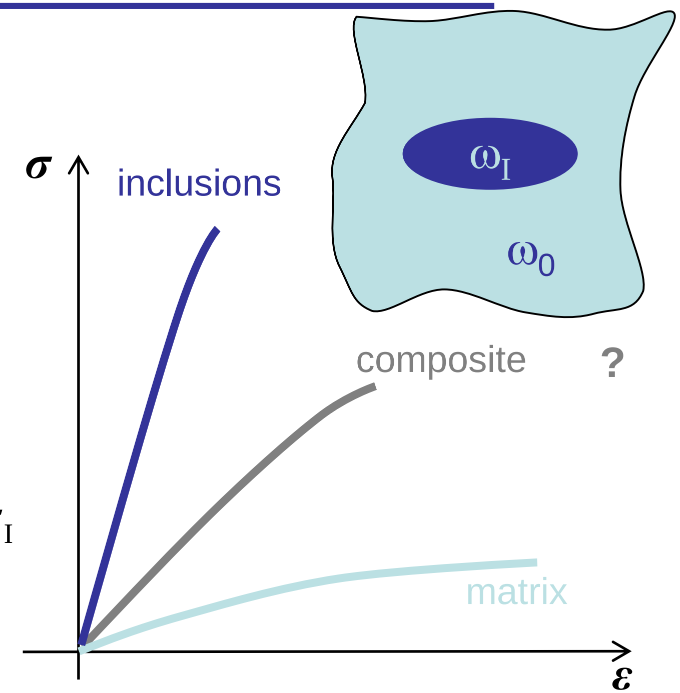
Mori and Tanaka 73, Hill 65, Ponte Castañeda 91, Suquet 95, Doghri et al 03, Lahellec et al. 11, Brassart et al. 12, …

- Key principles
  - Based on the averaging of the fields

$$\langle a \rangle = \frac{1}{V}\int_V a(\boldsymbol{X})\mathrm{d}V$$

  - Meso-response
    - From the volume ratios ($v_0 + v_\mathrm{I} = 1$)

$$\begin{cases} \bar{\boldsymbol{\sigma}} = \langle \boldsymbol{\sigma} \rangle = v_0 \langle \boldsymbol{\sigma} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\sigma} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\sigma}_0 + v_\mathrm{I} \boldsymbol{\sigma}_\mathrm{I} \\ \bar{\boldsymbol{\varepsilon}} = \langle \boldsymbol{\varepsilon} \rangle = v_0 \langle \boldsymbol{\varepsilon} \rangle_{\omega_0} + v_\mathrm{I} \langle \boldsymbol{\varepsilon} \rangle_{\omega_\mathrm{I}} = v_0 \boldsymbol{\varepsilon}_0 + v_\mathrm{I} \boldsymbol{\varepsilon}_\mathrm{I} \end{cases}$$



    - One more equation required

$$\boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0$$

  - Difficulty: find the adequate relations

$$\begin{cases} \boldsymbol{\sigma}_\mathrm{I} = f(\boldsymbol{\varepsilon}_\mathrm{I}) \\ \boldsymbol{\sigma}_0 = f(\boldsymbol{\varepsilon}_0) \\ \boldsymbol{\varepsilon}_\mathrm{I} = \boldsymbol{B}^{\varepsilon} : \boldsymbol{\varepsilon}_0 \end{cases} \qquad \boldsymbol{B}^{\varepsilon} \text{ ?}$$

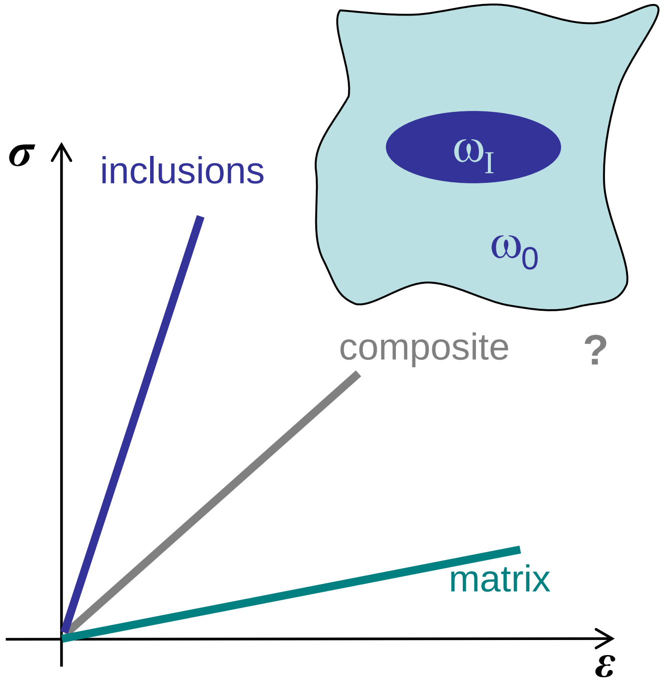
- Key principles (2)
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{0} = \overline{\boldsymbol{C}}_{0} : \boldsymbol{\varepsilon}_{0} \end{cases}$$

    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
    - Use Eshelby tensor

$$\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0, \overline{\boldsymbol{C}}_{\mathrm{I}}\right) : \boldsymbol{\varepsilon}_0$$

with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$



Mean-Field-Homogenization

- Key principles (2)
  - Linear materials
    - Materials behaviours

$$\begin{cases} \boldsymbol{\sigma}_{\mathrm{I}} = \overline{\boldsymbol{C}}_{\mathrm{I}} : \boldsymbol{\varepsilon}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{0} = \overline{\boldsymbol{C}}_{0} : \boldsymbol{\varepsilon}_{0} \end{cases}$$

    - Mori-Tanaka assumption $\boldsymbol{\varepsilon}^{\infty} = \boldsymbol{\varepsilon}_0$
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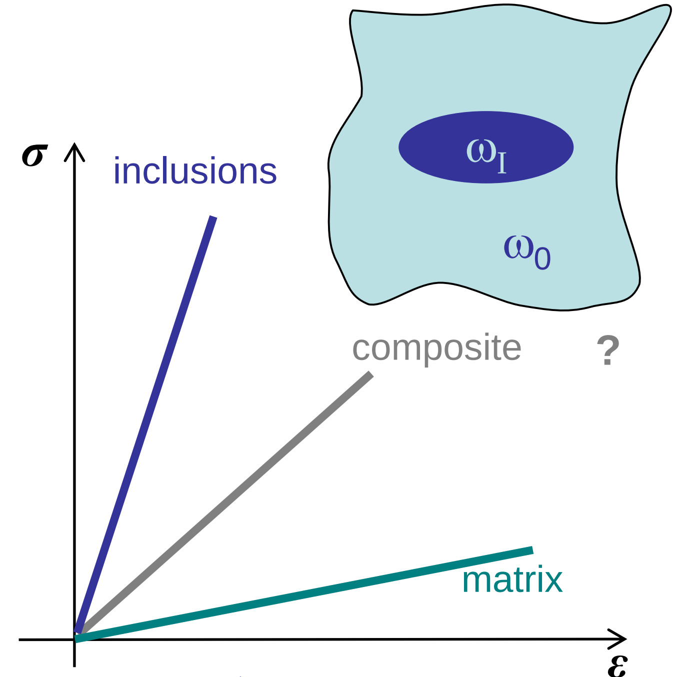
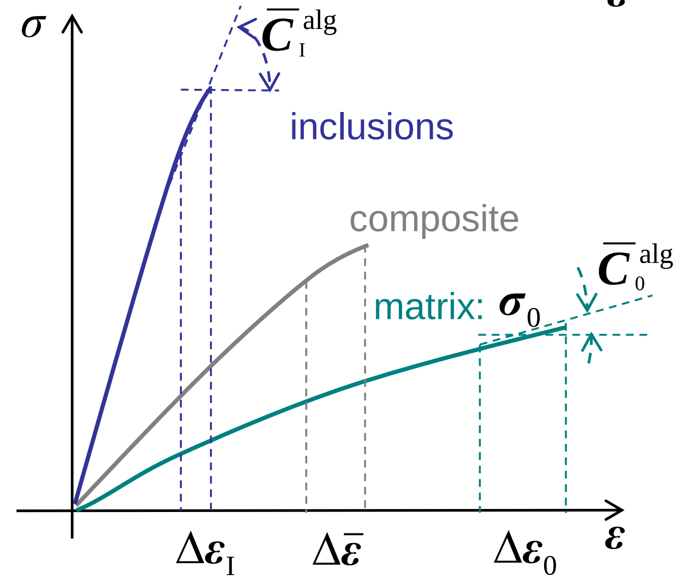
with $\boldsymbol{B}^{\varepsilon} = [\boldsymbol{I} + \boldsymbol{S} : \overline{\boldsymbol{C}}_0^{-1} : (\overline{\boldsymbol{C}}_1 - \overline{\boldsymbol{C}}_0)]^{-1}$

  - Non-linear materials
    - Define a Linear Comparison Composite (LCC)
    - Common approach: incremental tangent

$$\Delta\boldsymbol{\varepsilon}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}_0^{\,\mathrm{alg}}, \overline{\boldsymbol{C}}_{\mathrm{I}}^{\,\mathrm{alg}}\right) : \Delta\boldsymbol{\varepsilon}_0$$



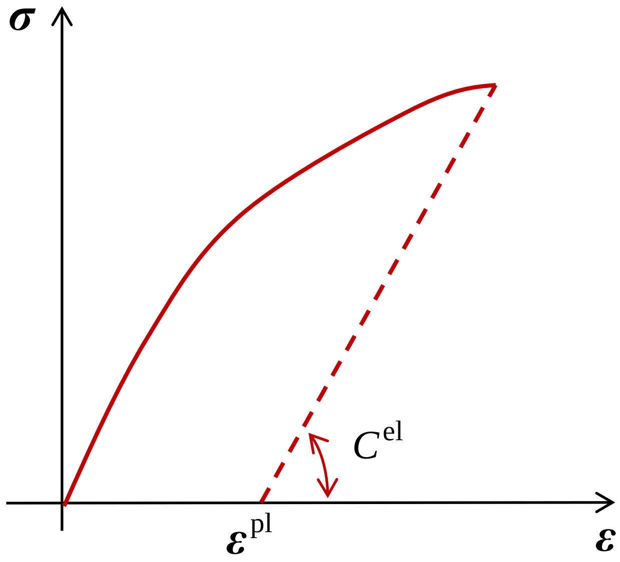


Content

- Micro-scale modelling
  - Incremental-Secant Mean-Field-Homogenization (MFH)
  - Damage-enhanced incremental-secant MFH

Incremental-secant mean-field-homogenization

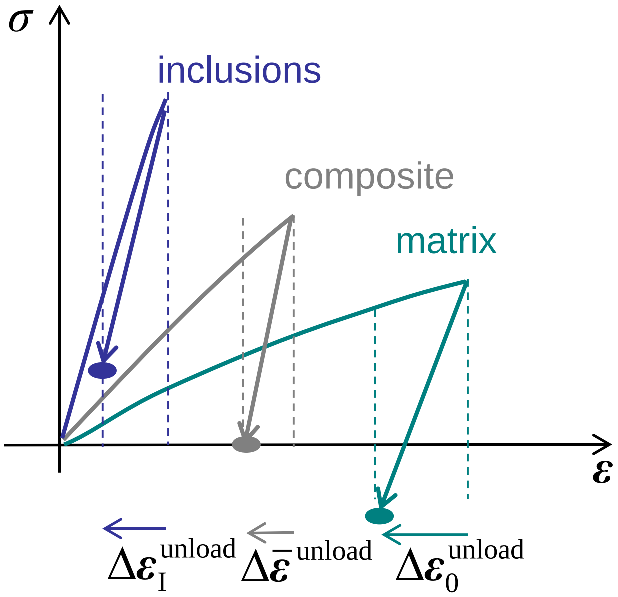
- Material model
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\mathrm{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\mathrm{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\mathrm{alg}} : \delta\boldsymbol{\varepsilon}$



Incremental-secant mean-field-homogenization

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

New Linear Comparison Composite (LCC)



Incremental-secant mean-field-homogenization

- New incremental-secant approach
  - Perform a virtual elastic unloading from previous solution
    - Composite material unloaded to reach the stress-free state
    - Residual stress in components

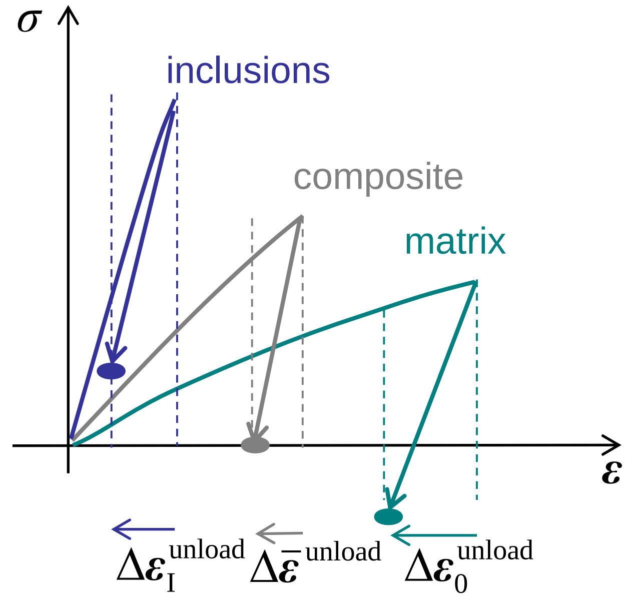
    New Linear Comparison Composite (LCC)

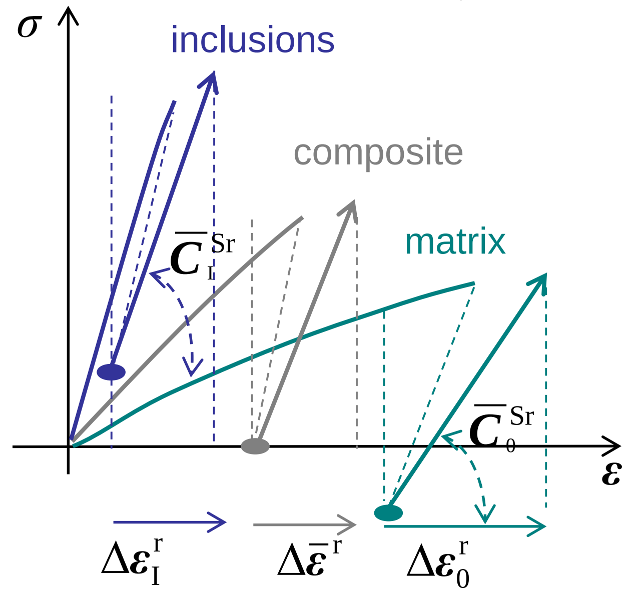
  - Apply MFH from unloaded state
    - New strain increments (>0)

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}/0} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}/0} + \Delta\boldsymbol{\varepsilon}^{\mathrm{unload}}_{\mathrm{I}/0}$$

    - Use of secant operators

$$\Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{\mathrm{I}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{0}, \overline{\boldsymbol{C}}^{\mathrm{Sr}}_{\mathrm{I}}\right) : \Delta\boldsymbol{\varepsilon}^{\mathrm{r}}_{0}$$



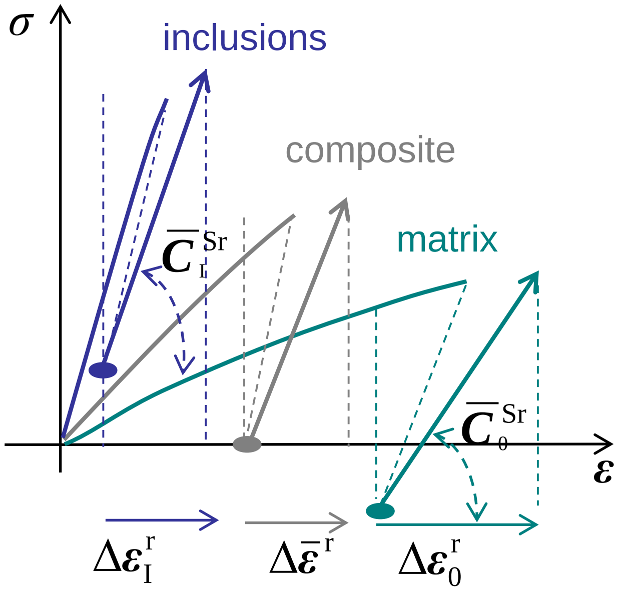


- New incremental-secant approach (2)
  - Equations summary
    - Inputs
      - Internal variables at last increment
      - Residual tensor after virtual unloading
      - $\Delta\bar{\boldsymbol{\varepsilon}}$ from FE resolution
    - Solve iteratively the system

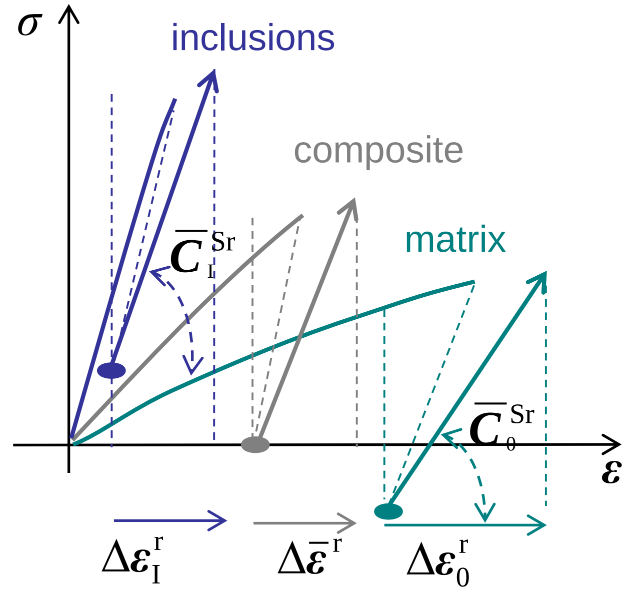
$$
\begin{cases}
\Delta\bar{\boldsymbol{\varepsilon}}^{\mathrm{r}} = \Delta\bar{\boldsymbol{\varepsilon}} + \Delta\bar{\boldsymbol{\varepsilon}}^{\mathrm{unload}} = v_0\Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} + v_{\mathrm{I}}\Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\
\Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} + \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{unload}} \\
\Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\
\Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, \bar{\boldsymbol{C}}_0^{\mathrm{Sr}}, \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}
\end{cases}
$$

    - With the stress tensors

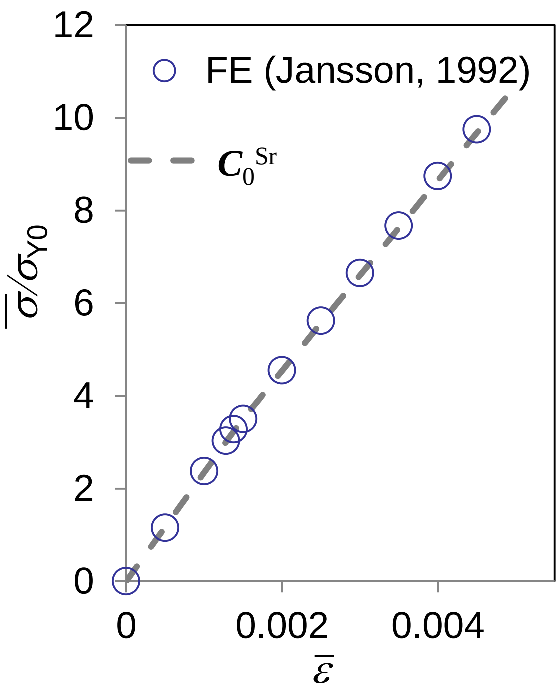
$$
\begin{cases}
\bar{\boldsymbol{\sigma}} = v_0\boldsymbol{\sigma}_0 + v_{\mathrm{I}}\boldsymbol{\sigma}_{\mathrm{I}} \\
\boldsymbol{\sigma}_{\mathrm{I}} = \boldsymbol{\sigma}_{\mathrm{I}}^{\mathrm{res}} + \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\
\boldsymbol{\sigma}_0 = \boldsymbol{\sigma}_0^{\mathrm{res}} + \bar{\boldsymbol{C}}_0^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}}
\end{cases}
$$



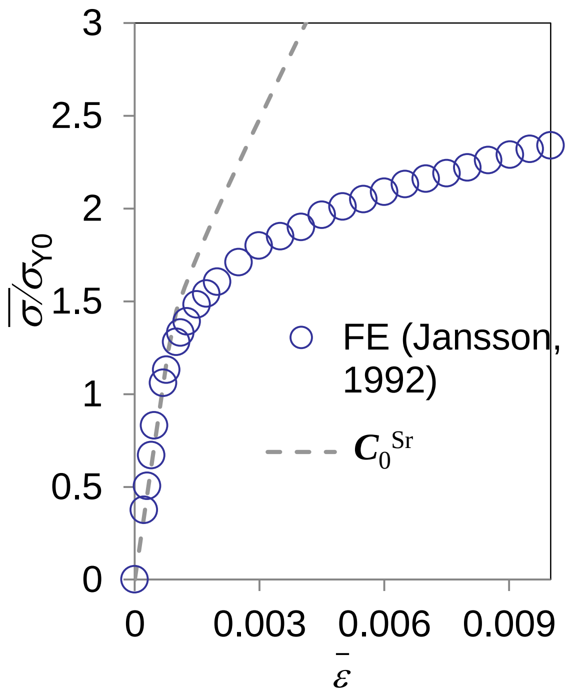
- Zero-incremental-secant method
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - For inclusions with high hardening (elastic)
    - Model is too stiff



Longitudinal tension



Transverse loading

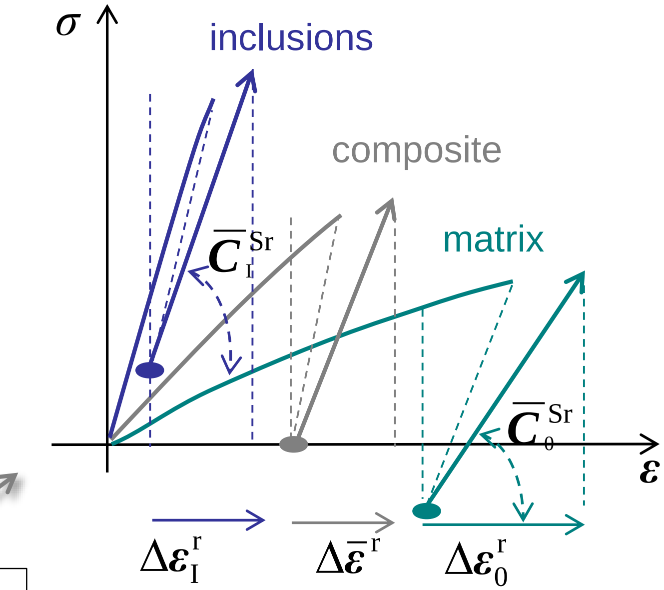


$$f(\boldsymbol{\sigma}, p) = \overline{\boldsymbol{\sigma}}^{eq} - \sigma^{Y} - R(\overline{p}) \leq 0$$

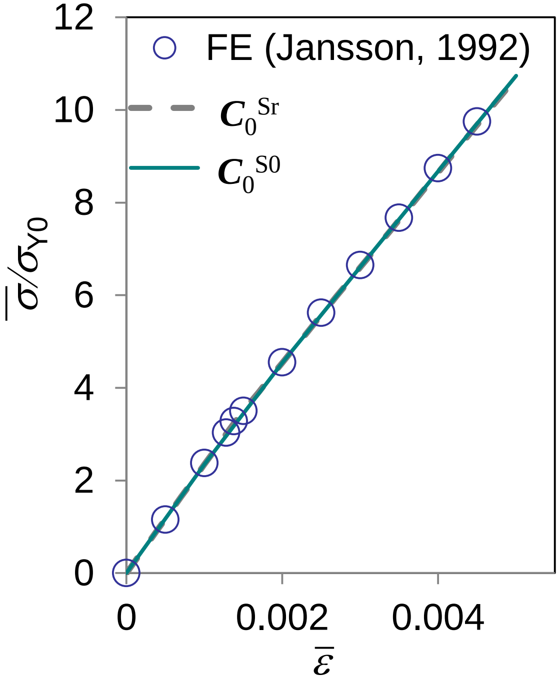
$\overline{\boldsymbol{\sigma}}^{eq}$ is <u>underestimated</u>

Incremental-secant mean-field-homogenization

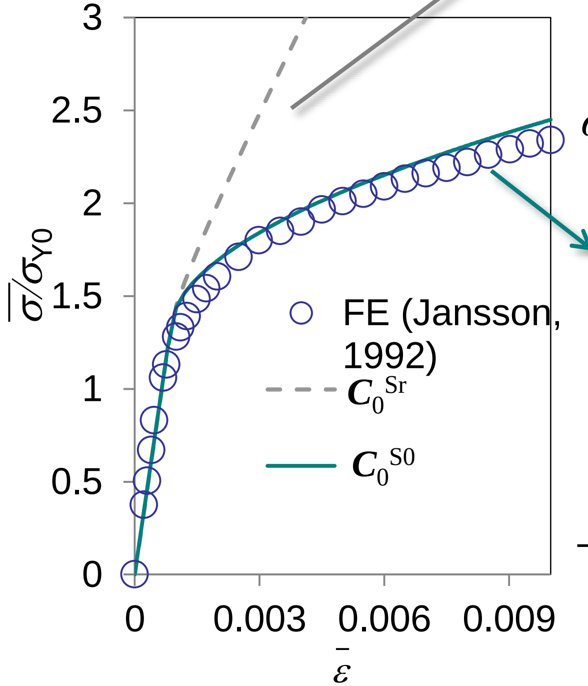
- Zero-incremental-secant method (2)
  - Continuous fibres
    - 55 % volume fraction
    - Elastic
  - Elasto-plastic matrix
  - Secant model in the matrix
    - Modified if negative residual stress

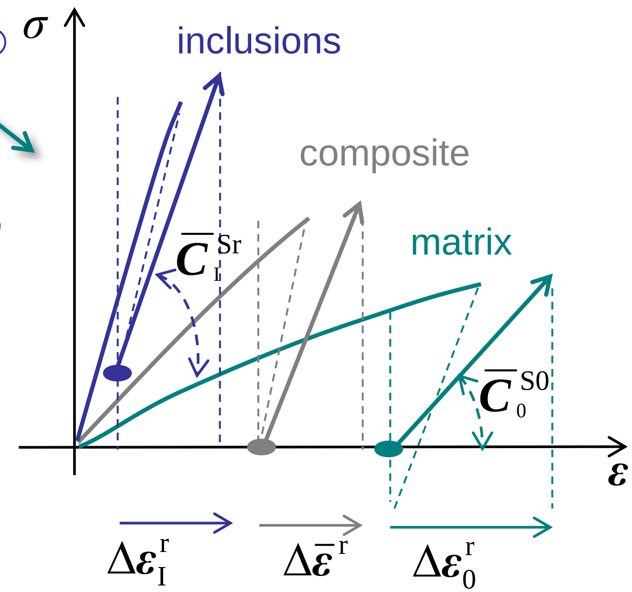


Longitudinal tension



Transverse loading





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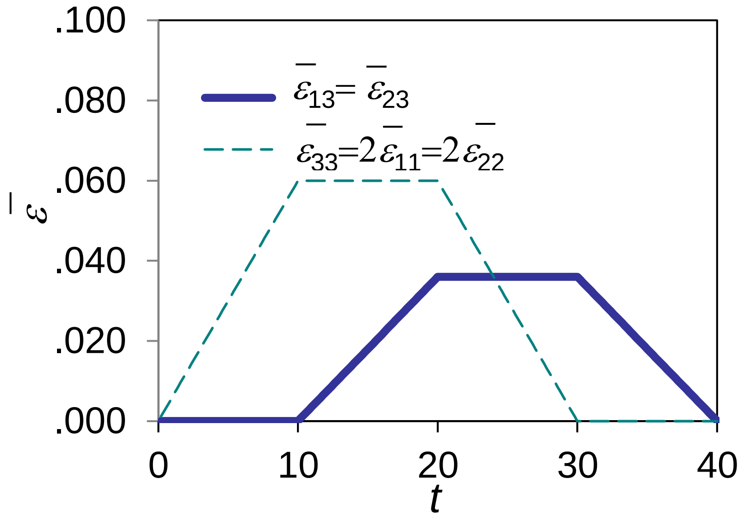
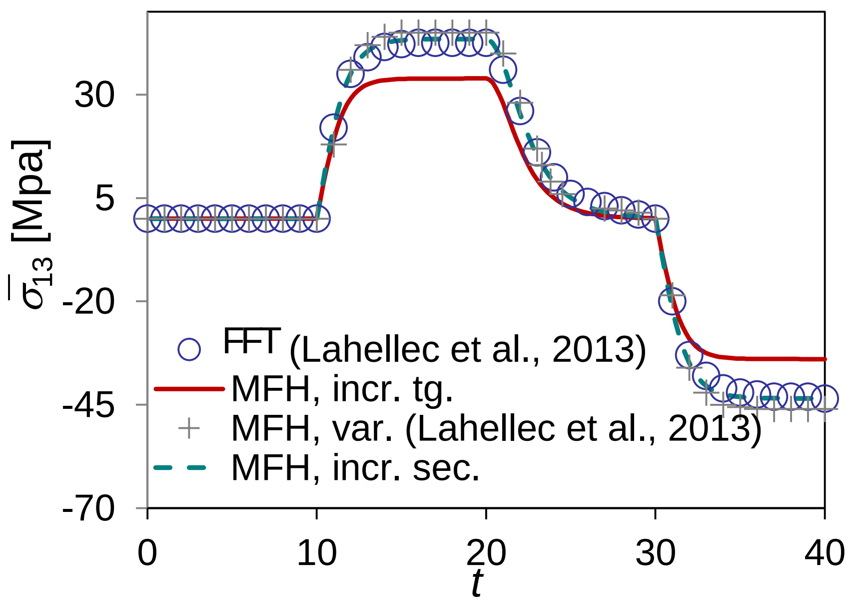
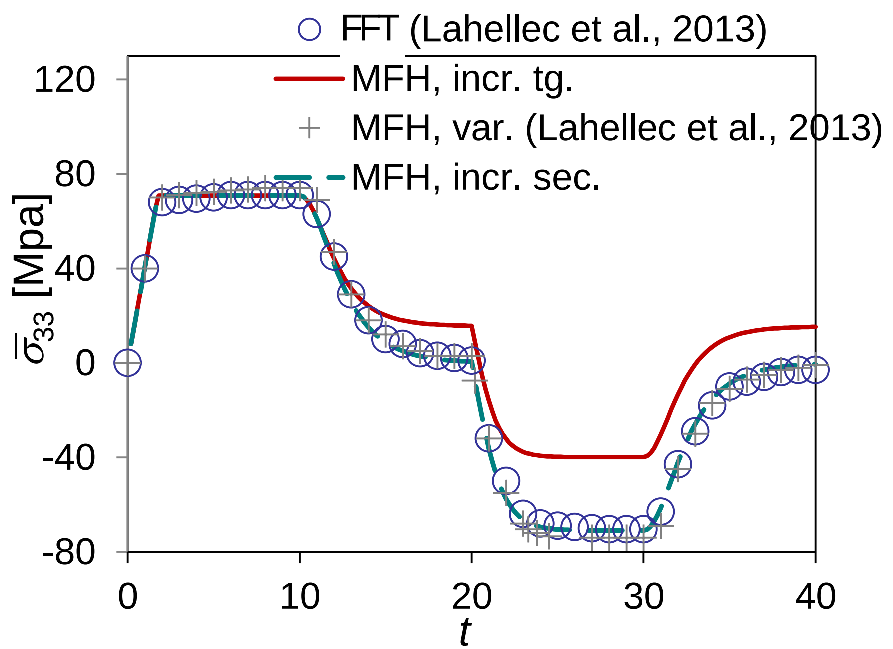
- Verification of the method
  - Spherical inclusions
    - 17 % volume fraction
    - Elastic
  - Elastic-perfectly-plastic matrix
  - Non-proportional loading

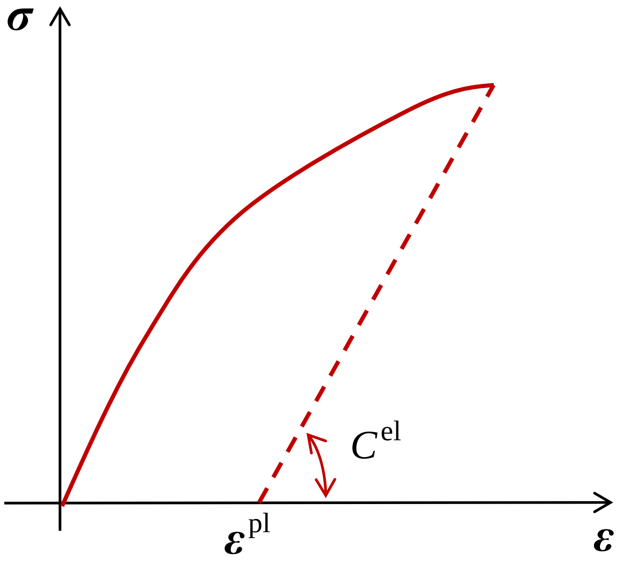






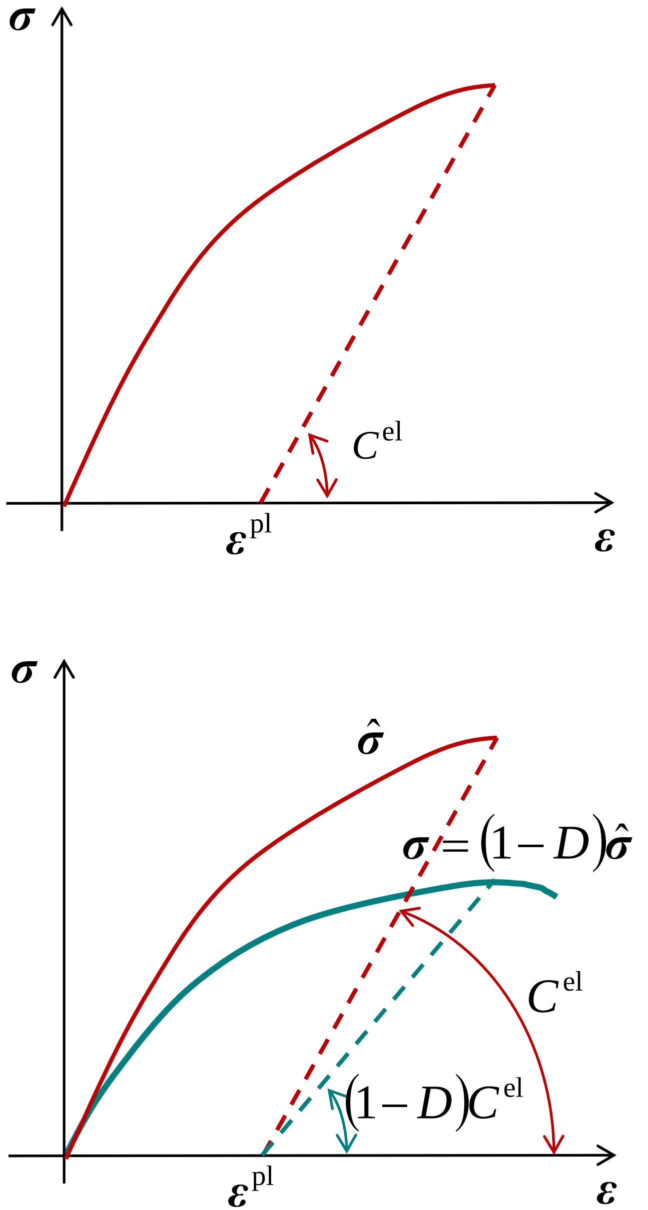
Non-local damage-enhanced MFH

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$



Non-local damage-enhanced MFH

- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
    - Plastic flow $\Delta\boldsymbol{\varepsilon}^{\text{pl}} = \Delta p \boldsymbol{N}$ & $\boldsymbol{N} = \dfrac{\partial f}{\partial \boldsymbol{\sigma}}$
    - Linearization $\delta\boldsymbol{\sigma} = \boldsymbol{C}^{\text{alg}} : \delta\boldsymbol{\varepsilon}$
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$



- Material models
  - Elasto-plastic material
    - Stress tensor $\boldsymbol{\sigma} = \boldsymbol{C}^{\text{el}} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\text{pl}})$
    - Yield surface $f(\boldsymbol{\sigma}, p) = \boldsymbol{\sigma}^{\text{eq}} - \sigma^{Y} - R(p) \leq 0$
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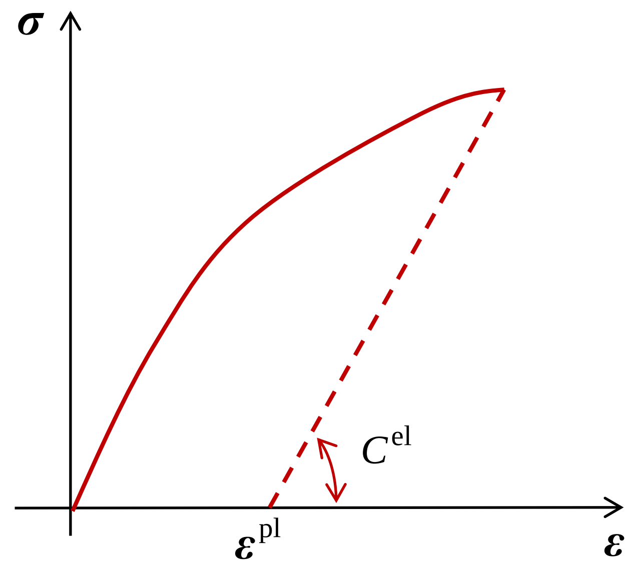
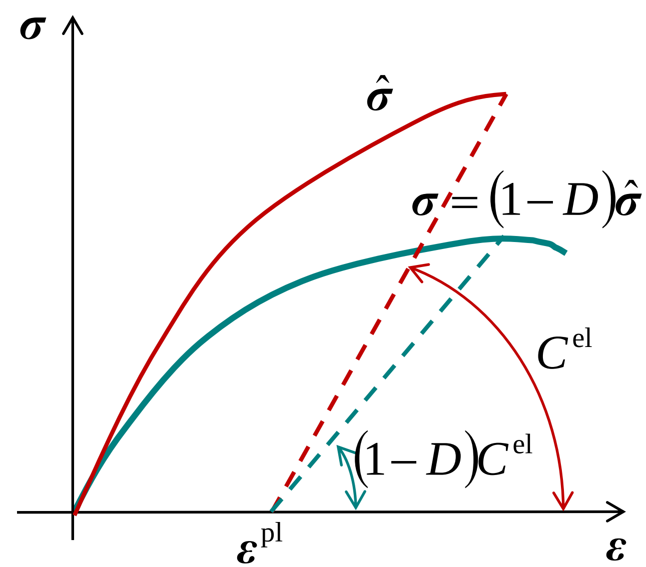
  - Local damage model
    - Apparent-effective stress tensors $\boldsymbol{\sigma} = (1 - D)\hat{\boldsymbol{\sigma}}$
    - Plastic flow in the effective stress space
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta p)$
  - Non-Local damage model
    - Damage evolution $\Delta D = F_D(\boldsymbol{\varepsilon}, \Delta \tilde{p})$
    - Anisotropic governing equation $\tilde{p} - \nabla \cdot (\boldsymbol{c}_{\text{g}} \cdot \nabla \tilde{p}) = p$
    - Linearization

$$\delta\boldsymbol{\sigma} = \left[(1 - D)\boldsymbol{C}^{\text{alg}} - \hat{\boldsymbol{\sigma}} \otimes \frac{\partial F_D}{\partial \boldsymbol{\varepsilon}}\right] : \delta\boldsymbol{\varepsilon} - \hat{\boldsymbol{\sigma}} \frac{\partial F_D}{\partial \tilde{p}} \delta\tilde{p}$$



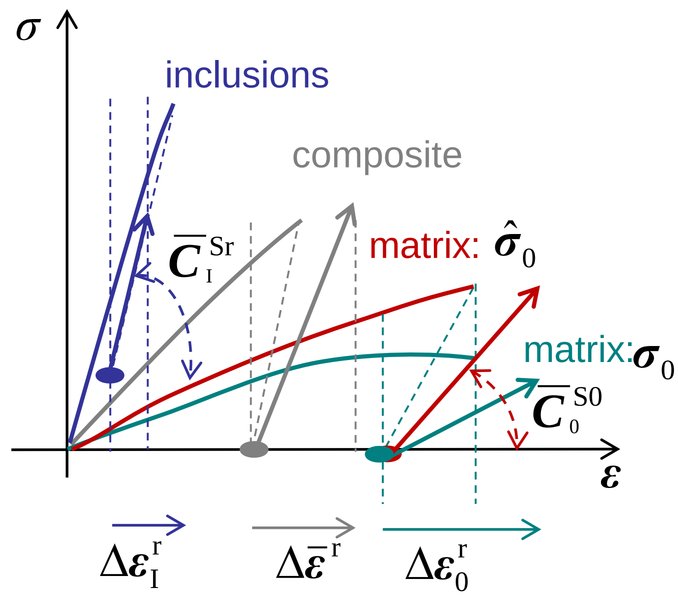


- Equations summary: zero-approach
  - For soft matrix response
    - Remove residual stress in matrix
    - Avoid adding spurious internal energy
  - Solve iteratively the system

$$\begin{cases} \Delta\bar{\boldsymbol{\varepsilon}}^{(\mathrm{r})} = v_0 \Delta\boldsymbol{\varepsilon}_0^{(\mathrm{r})} + v_{\mathrm{I}} \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{(\mathrm{r})} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_{\mathrm{I}} + \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} = \Delta\boldsymbol{\varepsilon}_0 + \Delta\boldsymbol{\varepsilon}_0^{\mathrm{unload}} \\ \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} = \boldsymbol{B}^{\varepsilon}\left(\mathrm{I}, (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{S0}}, \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}}\right) : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$

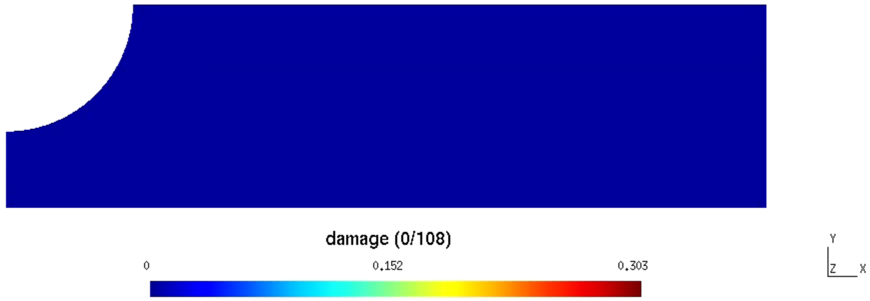
  - With the stress tensors

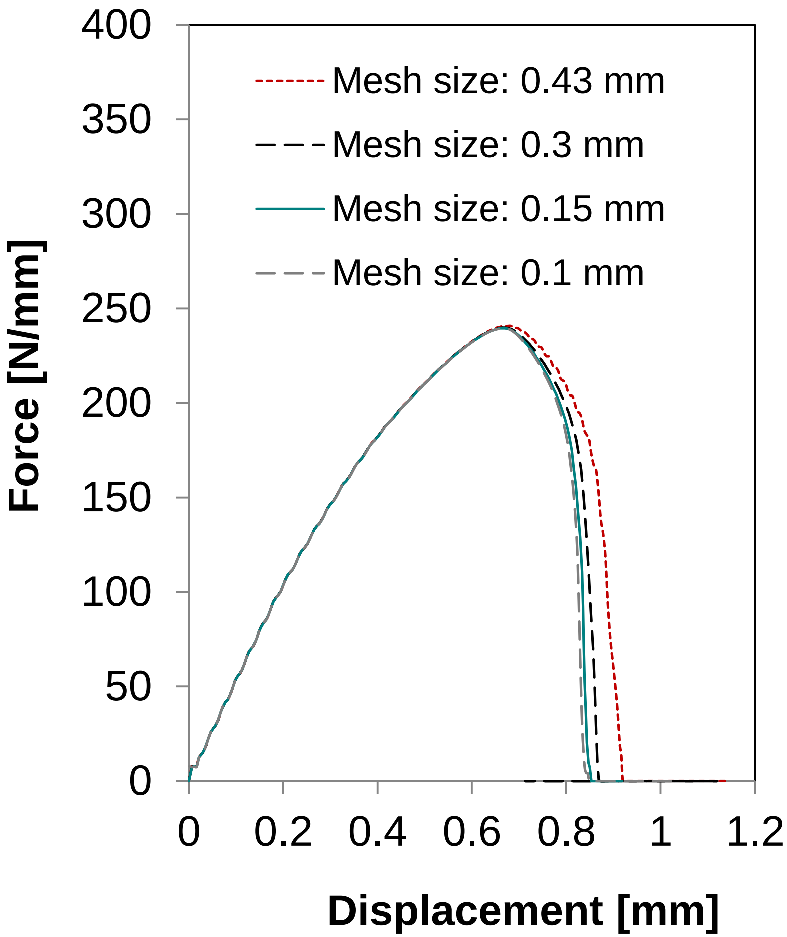
$$\begin{cases} \bar{\boldsymbol{\sigma}} = v_0 \boldsymbol{\sigma}_0 + v_{\mathrm{I}} \boldsymbol{\sigma}_{\mathrm{I}} \\ \boldsymbol{\sigma}_{\mathrm{I}} = \boldsymbol{\sigma}_{\mathrm{I}}^{\mathrm{res}} + \bar{\boldsymbol{C}}_{\mathrm{I}}^{\mathrm{Sr}} : \Delta\boldsymbol{\varepsilon}_{\mathrm{I}}^{\mathrm{r}} \\ \boldsymbol{\sigma}_0 = (1-D)\bar{\boldsymbol{C}}_0^{\mathrm{S0}} : \Delta\boldsymbol{\varepsilon}_0^{\mathrm{r}} \end{cases}$$



Non-local damage-enhanced MFH

- Mesh-size effect
  - Fictitious composite
    - 30%-UD fibres
    - Elasto-plastic matrix with damage
  - Notched ply





Content

- Multi-scale method for the failure analysis of composite laminates
  - Intra-laminar failure: Non-local damage-enhanced mean-field-homogenization
  - Inter-laminar failure: Hybrid DG/cohesive zone model
  - Experimental validation

- Weak formulation of a composite laminate
  - Strong form

$$\begin{cases} \nabla \cdot \bar{\boldsymbol{\sigma}}^T + \boldsymbol{f} = \boldsymbol{0} & \text{for each homogenized ply } \Omega_i \\ \tilde{p} - \nabla \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p} \right) = p & \text{for the matrix phase} \end{cases}$$

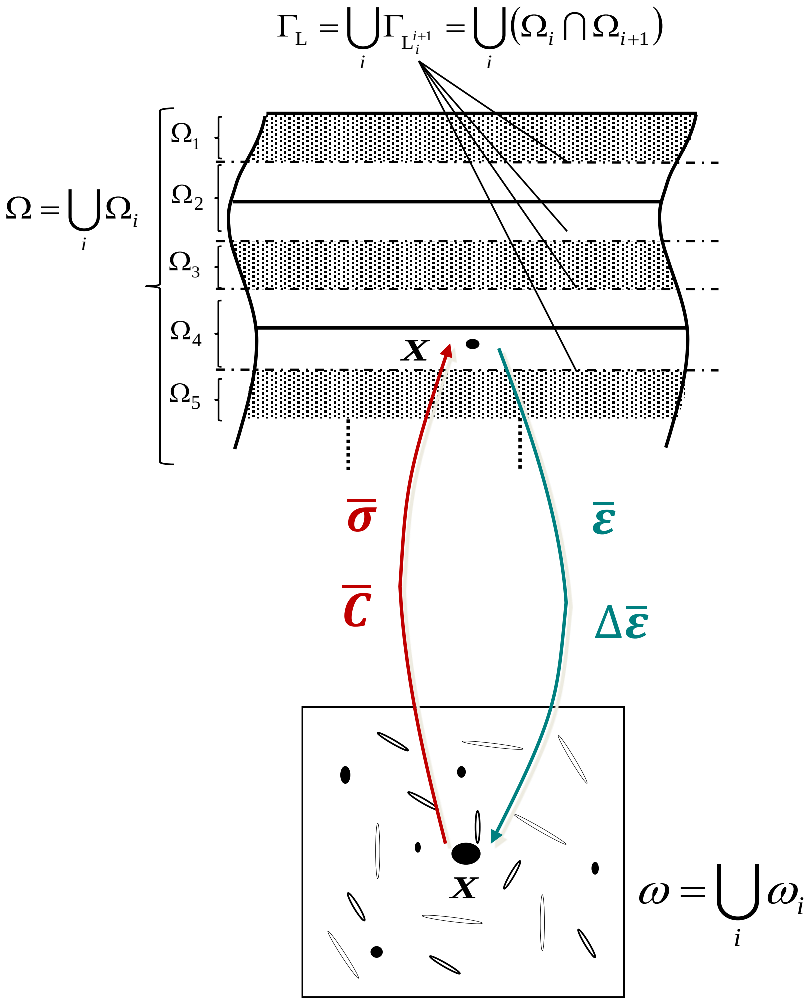
  - Boundary conditions

$$\begin{cases} \bar{\boldsymbol{\sigma}} \cdot \bar{\boldsymbol{n}} = \bar{\boldsymbol{T}} \\ \bar{\boldsymbol{n}} \cdot \left(\boldsymbol{c}_{\mathrm{g}} \cdot \nabla \tilde{p} \right) = 0 \end{cases}$$

  - Macro-scale finite-element discretization

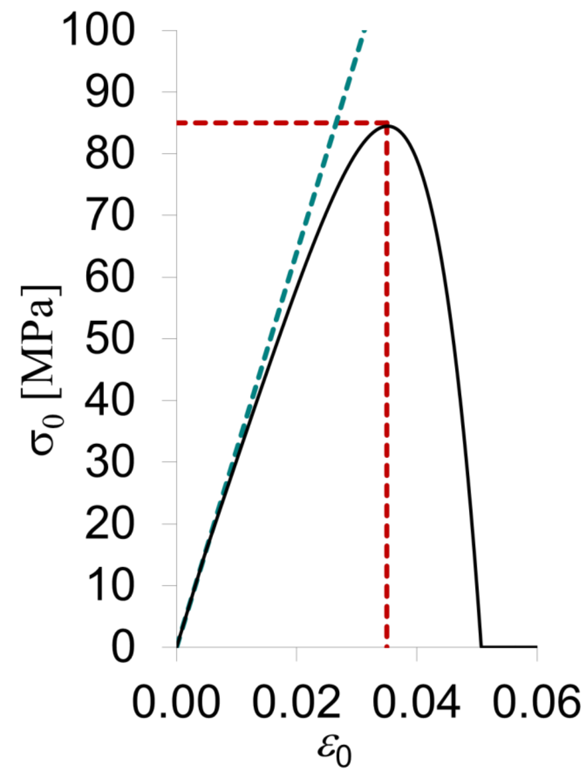
$$\begin{cases} \tilde{p} = N^a_{\tilde{p}} \tilde{\mathbf{p}}^a \\ \bar{\boldsymbol{u}} = N^a_u \bar{\boldsymbol{u}}^a \end{cases}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{K}_{\bar{u}\bar{u}} & \boldsymbol{K}_{\bar{u}\tilde{p}} \\ \boldsymbol{K}_{\tilde{p}\bar{u}} & \boldsymbol{K}_{\tilde{p}\tilde{p}} \end{bmatrix} \begin{bmatrix} d\bar{\boldsymbol{u}} \\ d\tilde{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{\mathrm{ext}} - \boldsymbol{F}_{\mathrm{int}} \\ \boldsymbol{F}_p - \boldsymbol{F}_{\tilde{p}} \end{bmatrix}$$

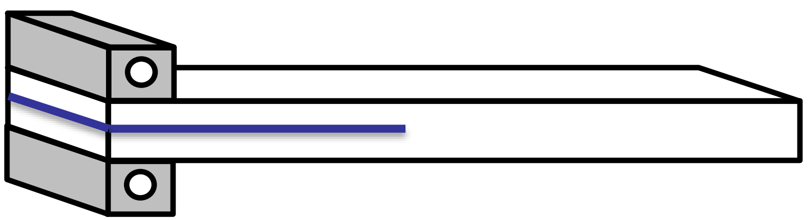


Experimental validation

- 60%-UD Carbon-fibres reinforced epoxy
  - Carbon fibres
    - Use of transverse isotropic elastic material
  - Elasto-plastic matrix with damage
    - Use manufacturer Young's modulus
    - Use manufacturer strength

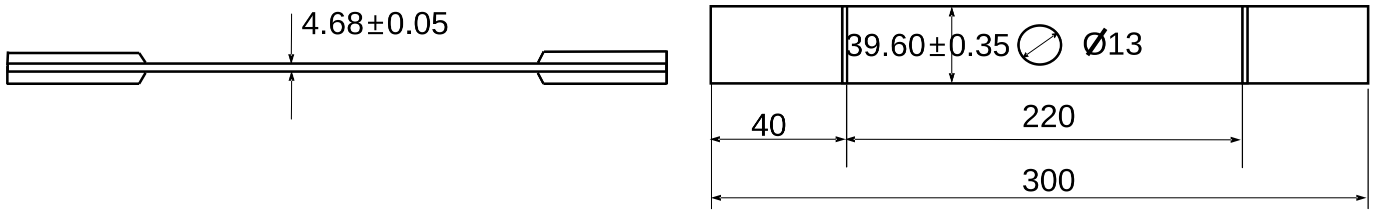


- Delamination: Double Cantilever Beam
  - Critical energy realise rates
    - G_{IC} = 600 J/m^2
    - G_{IIC} = 1200 J/m^2 (assumption)

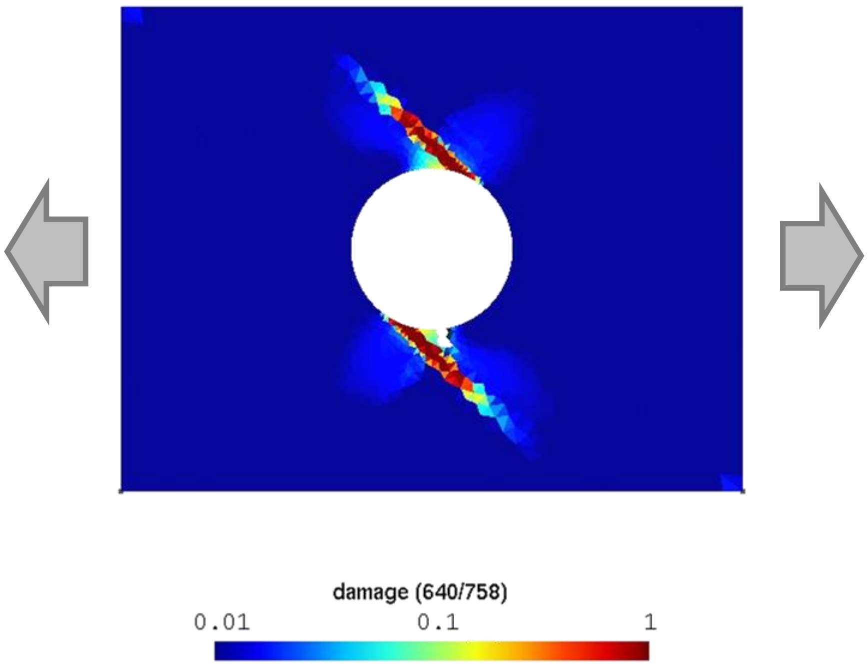


Experimental validation

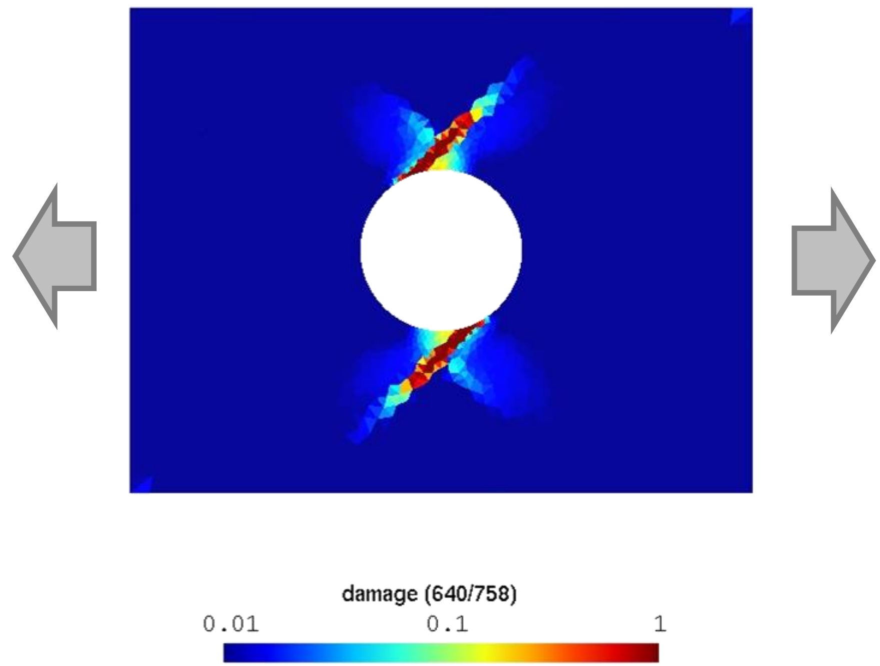
- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate
  - Tensile test on several coupons



  - Propagation of the damaged zones in agreement with the fibre direction



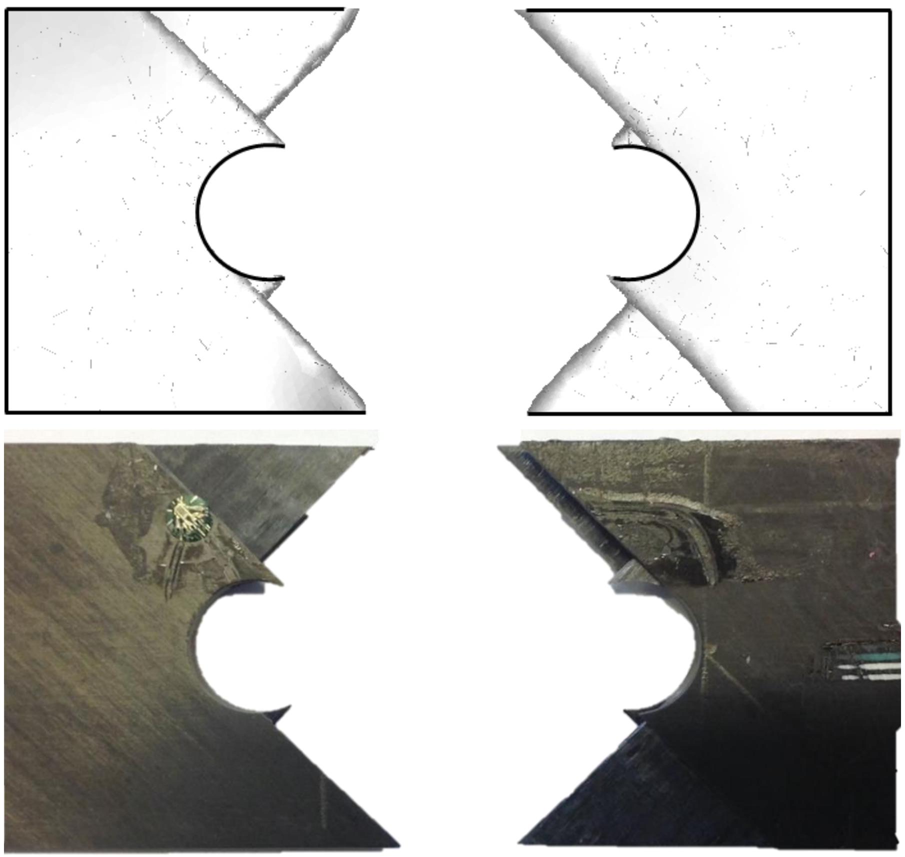
-45º-ply

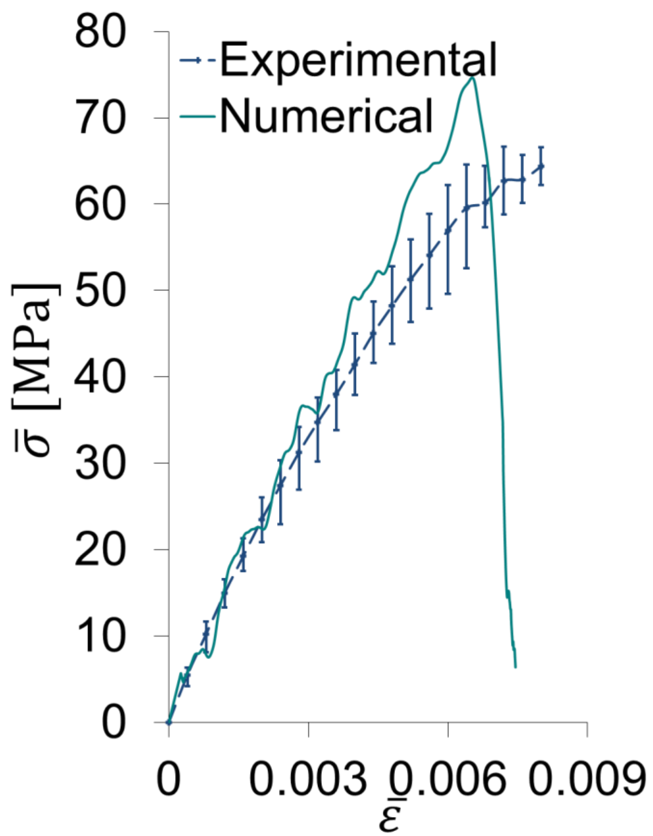


45º-ply

Experimental validation

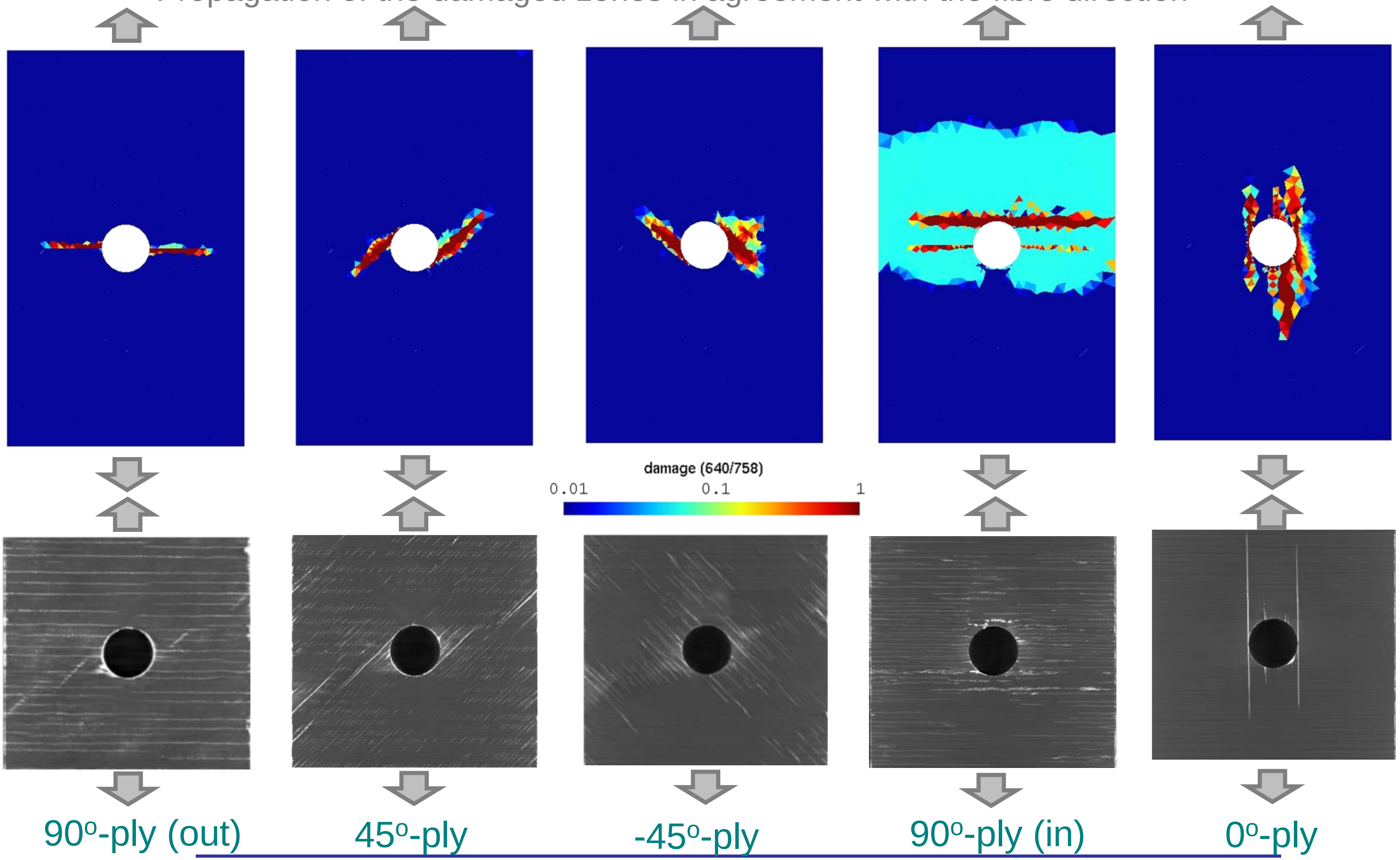
- $[45^o{}_4 / -45^o{}_4]_S$- open hole laminate (2)
  - Predicted delamination zones in agreement with experiments
  - Tensile stress within 15 %





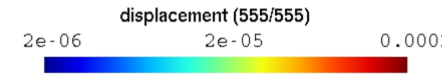
Experimental validation

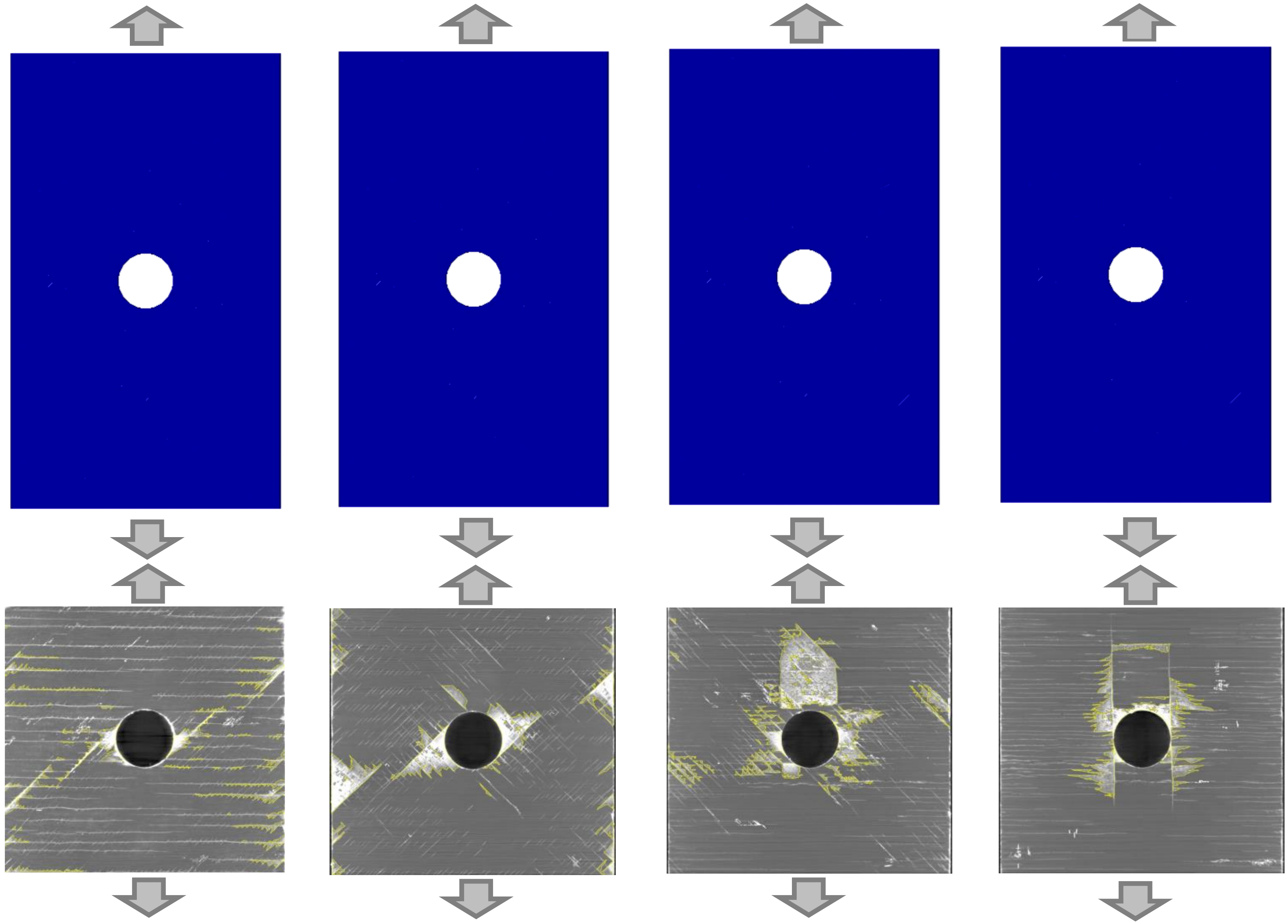
- $[90^o / 45^o / -45^o / 90^o / 0^o]_S$- open hole laminate (2)
  - Propagation of the damaged zones in agreement with the fibre direction



Experimental validation

- $[90^{o} / 45^{o} / -45^{o} / 90^{o} / 0^{o}]_S$- open hole laminate (3)
  - Predicted delamination zones in agreement with experiments





90^{o} (out) / 45^{o} | 45^{o} / -45^{o} | -45^{o} / 90^{o} (in) | 90^{o} (in) / 0^{o}

Conclusions

- Multi-scale method for the failure analysis of composite laminates
  - Damage-enhanced MFH
  - Non-local implicit formulation
  - Hybrid DG/CZM for delamination

- Experimental validation
  - Open-hole laminates
  - Different stacking sequences